

Complexity classification of quantum many-body systems according to the Pair of Order-Disorder Indices (PODI).

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The *statistical* measures of complexity $C(N)$ defined by López-Ruiz, Mancini, and Calbet (LMC) and $\Gamma_{\alpha,\beta}(N)$ according to Shiner, Davison and Landsberg (SDL) are calculated as functions of the number of particles N for four quantum many-body systems, i.e. atoms, nuclei, atomic clusters, and correlated atoms in a trap (bosons). The strengths of *disorder* α and order β are evaluated for each system by imposing the condition $\Gamma_{\alpha,\beta}(N) = C(N)$. The proper pair (α, β) , obtained by the above requirement, can serve as a Pair of Order-Disorder Indices (PODI), characterizing quantitatively order versus disorder in any quantum system. According to the above classification, we assign to bosons the complexity character of *disorder*, to atoms the character of *order*, while nuclei and atomic clusters are (less) *disordered* and lie between them. This criterion can be used to estimate the relative contribution of order and disorder to complexity for other more complicated quantum systems as well and even classical ones, if one is able to describe them probabilistically. We also address the issue, whether those systems can grow in complexity as N increases. Our comparative study indicates that atoms are the only quantum system out of four, which is ordered, with the ability to self-organize. We conjecture that this is an information-theoretic reason that atoms are suitable as building blocks of larger structures of biological interest i.e. molecules and macromolecules. This is in the spirit of Wheeler's *it from bit* quote, the project to present everything about the Universe in terms of information theory.

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I. INTRODUCTION

The question whether physical or biological systems can organize themselves without the intervention of an external factor, is a hot subject in the community of scientists interested in complexity. A practical way to answer such a question is to use a suitable definition of complexity and check if this quantity increases with the number of particles N , implying the ability or inability to self-organize. There are several measures of complexity in the literature. One of them is the algorithmic complexity according to Kolmogorov and Chaitin [1, 2] defined as the length of the shortest possible program necessary to reproduce a given object. The fact that a given program is indeed the shortest one, is hard to prove. In this paper we employ the *statistical* measure of complexity C , defined by López-Ruiz, Mancini, and Calbet (LMC) [3], which can be calculated easily, provided that the information content of a quantum system is known from previous work [4]-[14]. In the same spirit, we use the measure of complexity $\Gamma_{\alpha,\beta}$ according to Shiner, Davison and Landsberg (SDL) [15].

We intend to calculate and compare $C(N)$ with $\Gamma_{\alpha,\beta}(N)$ as functions of the number of particles N in four quantum systems, namely atomic nuclei, atoms, atomic clusters, and correlated atoms in a trap–bosons. Here, we use continuous probability density distributions of particles of quantum systems, while in [16] we obtained (α, β) using $C(Z)$ and $\Gamma_{\alpha,\beta}(Z)$, taking into account discrete probability distributions of electron configurations in atoms.

It is noted that such complexity measures, based in a probabilistic description of a quantum system, have been calculated quantitatively (as functions of Z) for the first time in [13, 14], using atoms as a test bed. A previous preliminary step was the finding that Landsberg’s order parameter [17] is an increasing function of N for fermions and bosonic systems [9] (see also [18]).

The aim of the present work is to find specific overall values of (α, β) for each system, by requiring that approximately $\Gamma_{\alpha,\beta}(N) = C(N)$. We investigate the possibility whether the pair (α, β) , chosen in such a way (PODI: Pair of Order-Disorder Indices) is useful for the classification of quantum many-body systems according to the contribution of order and disorder to complexity.

Our paper is organized as follows: In Sec. II we define measures of information and

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complexity, in Sec. III we present our numerical results and Sec. IV contains our conclusions.

II. MEASURES OF INFORMATION CONTENT AND COMPLEXITY OF A SYSTEM

Shannon's information entropy [19], is defined as

$$S_r = - \int \rho(\mathbf{r}) \ln \rho(\mathbf{r}) d\mathbf{r} \quad (1)$$

$$S_k = - \int n(\mathbf{k}) \ln n(\mathbf{k}) d\mathbf{k} \quad (2)$$

where $\rho(\mathbf{r})$ and $n(\mathbf{k})$ are normalized to one density distributions in position and momentum spaces respectively. S_r, S_k depend on the units used to measure \mathbf{r} and \mathbf{k} , while the important quantity is the sum $S = S_r + S_k$, which is scale invariant i.e. independent of units. There is a delicate balance between conjugate spaces leading to the following results:

First, the entropic uncertainty relation (EUR) holds of the form $S = S_r + S_k \geq 3(1 + \ln \pi)$ for a 3-dimensional quantum system [20]. The lower bound is attained for a Gaussian distribution. The above inequality is stronger than the Heisenberg uncertainty relation, because the right-hand side of EUR does not depend on the state, while Heisenberg's does depend. Also Heisenberg's inequality can be derived from EUR, while the inverse is not true. S represents the information content of the system in bits, if one uses logarithm with base 2 and in nats (natural unit of information), if the logarithm is natural.

Second, the universal property $S = a + b \ln N$ was proposed and verified for various quantum systems (nuclei, atoms, atomic clusters and correlated atoms in a trap) [10], [11]. The parameters a, b depend on the system under consideration. That property holds for systems of various sizes, obeying different statistics (fermions or bosons) with different numbers of particles and various interactions. Specifically, the sizes of those systems range from the order of fermis (10^{-13} cm) in nuclei, to 10^4 Å (10^{-4} cm) for bosonic systems, while the number of particles N starts from a few to a hundred and goes up to millions (bosons).

The Shiner, Davison and Landsberg (SDL) measure complexity $\Gamma_{\alpha,\beta}$ is defined as:

$$\Gamma_{\alpha,\beta} = \Delta^\alpha \cdot \Omega^\beta = \Delta^\alpha (1 - \Delta)^\beta \quad (3)$$

where

$$\Delta = \frac{S}{S_{\max}} \quad \text{and} \quad \Omega = 1 - \Delta \quad (4)$$

are the normalized measures of disorder and order respectively, according to Landsberg [17], obeying the inequalities $0 < \Delta < 1, 0 < \Omega < 1$.

The parameter α represents the strength of disorder , while β the strength of order. We have a complexity measure $\Gamma_{\alpha,\beta}$ of category I, if $\beta = 0$ and $\alpha > 0$, where complexity is an increasing function of disorder. In category II ($\alpha > 0, \beta > 0$) $\Gamma_{\alpha,\beta}$ is a convex function, while in category III ($\alpha = 0, \beta > 0$) $\Gamma_{\alpha,\beta}$ is a decreasing function of disorder Δ .

The LMC measure C [3] is defined as

$$C = S \cdot D \quad (5)$$

where S denotes the information content stored in the system (in our case Shannon's information entropy sum $S = S_r + S_k$ [19]) and D is the disequilibrium of the system.

For a discrete probability distribution $\{p_i\}$, the disequilibrium D can be defined as the quadratic distance of $\{p_i\}$ to the equiprobable (uniform) state $p_i = \frac{1}{n}, i = 1, 2, \dots$. Thus

$$D = \sum_{i=1}^k \left(p_i - \frac{1}{n} \right)^2$$

while for a continuous probability distribution $\rho(\mathbf{r})$, it is extended in position-space as

$$D_r = \int \rho^2(\mathbf{r}) d(\mathbf{r})$$

and in momentum-space

$$D_k = \int n^2(\mathbf{k}) d(\mathbf{k})$$

The extension from the discrete to the continuous case is justified in [9]. The proper combination of D_r and D_k , to be inserted into (1) is

$$D = D_r \cdot D_k \quad (6)$$

In fact D_r has the dimension of inverse volume, while D_k of volume, leading to the dimensionless quantity D .

III. NUMERICAL RESULTS

We intend to calculate numerically the LMC measure of complexity $C(N) = S \cdot D$ and the SDL one, $\Gamma_{\alpha,\beta}(N) = \Delta^\alpha \cdot \Omega^\beta$ as functions of the number of particles N (electrons in

atoms, nucleons in nuclei, valence electrons in atomic clusters, and alkali atoms in bosonic traps). Our calculations are facilitated by our previous research in information entropy S . The key quantities of our calculations are the density distributions in position-space $\rho(\mathbf{r})$ and momentum-space $n(\mathbf{k})$, obtained as follows.

For nuclei we performed microscopic mean-field calculations [10], using a density-dependent Skyrme force, in particular the SKIII force [21]. For atoms we used the RHF (Roothaan-Hartree-Fock) electron distributions [22], applied to calculations of atomic complexity [13], [14]. For atomic clusters, we employed the jellium model (Ekcard model) [10], [23], [24], while for bosons in a trap we carried out a numerical calculation of densities in both position- and momentum-spaces, solving numerically the non-linear Gross-Pitaevski equation [9]. In all of the above calculations special care has been devoted to an accurate treatment of the Fourier transform of $\rho(\mathbf{r})$ in order to obtain $n(\mathbf{k})$ for each system.

Next, the distributions $\rho(\mathbf{r})$ and $n(\mathbf{k})$ calculated as described above, are inserted into relations (1) and (2) giving $S_r(N)$ and $S_k(N)$ for each system, needed to find the sum $S(N) = S_r(N) + S_k(N)$. Thus, $C(N) = S(N) \cdot D(N)$ can be obtained easily. However, in order to evaluate $\Gamma_{\alpha,\beta}(N) = \Delta(N)^\alpha \cdot (1 - \Delta(N))^\beta$, we need in addition $\Delta(N) = S(N)/S_{\max}(N)$, where $S_{\max}(N) = S_{r\max}(N) + S_{k\max}(N)$.

For discrete probability distributions the value of S_{\max} , is attained for an equiprobable (uniform) density distribution i.e. $S_{\max} = \ln N$, where N is the number of states with probabilities $\{p_i\}$, $i = 1, 2, \dots, N$. S is minimum, $S_{\min} = 0$, when only one probability is different from 0, specifically $p_i = 1$ for a fixed i , while all the others vanish. For continuous probability distributions, calculations for S_{\min} and S_{\max} are based to the following inequalities, introduced for the first time and verified for atoms by Gadre and collaborators [6], [7], for atomic clusters and nuclei in [11] and bosonic systems (correlated alkali atoms in a trap) in [9] (see also [25]).

$$S_{r\min} \leq S_r \leq S_{r\max} \quad (7)$$

$$S_{k\min} \leq S_k \leq S_{k\max} \quad (8)$$

$$S_{\min} \leq S \leq S_{\max} \quad (9)$$

where $S = S_r + S_k$ and $\rho(\mathbf{r})$, $n(\mathbf{k})$ are normalized to 1, i.e. $\int \rho(\mathbf{r}) d\mathbf{r} = 1$ and

$$\int n(\mathbf{k}) d\mathbf{k} = 1.$$

The lower and upper limits for density distributions normalized to one, are:

$$\begin{aligned} S_{r\min} &= \frac{3}{2} (1 + \ln \pi) - \frac{3}{2} \ln \left(\frac{4}{3} T \right) \\ S_{r\max} &= \frac{3}{2} (1 + \ln \pi) + \frac{3}{2} \ln \left(\frac{2}{3} \langle r^2 \rangle \right) \end{aligned} \quad (10)$$

and

$$\begin{aligned} S_{k\min} &= \frac{3}{2} (1 + \ln \pi) - \frac{3}{2} \ln \left(\frac{2}{3} \langle r^2 \rangle \right) \\ S_{r\max} &= \frac{3}{2} (1 + \ln \pi) + \frac{3}{2} \ln \left(\frac{4}{3} T \right) \end{aligned} \quad (11)$$

Thus

$$S_{\min} = 3 (1 + \ln \pi) - \frac{3}{2} \ln \left(\frac{8}{9} \langle r^2 \rangle T \right) \quad (12)$$

$$S_{\max} = 3 (1 + \ln \pi) + \frac{3}{2} \ln \left(\frac{8}{9} \langle r^2 \rangle T \right) \quad (13)$$

where $\langle r^2 \rangle$ is the mean square radius of the system and T its kinetic energy. We use, as an input in $\Delta = \frac{S}{S_{\max}}$, the values of S_{\max} according to relation (13).

We repeated and verified the above inequalities for atoms in [13], [14] and next, we checked their validity numerically for nuclei, atomic clusters and bosonic traps in [9], [10]. We note that $\langle r^2 \rangle$ and T , can be calculated employing $\rho(\mathbf{r})$ and $n(\mathbf{k})$. Thus, we obtain the function $S_{\max}(N)$ for each quantum system.

Following the procedure described above, we calculate and plot the functions $S(N)$, $S_{\max}(N)$, $C(N)$ and $\Gamma_{\alpha,\beta}(N)$ (for various values (α, β)), and $D(N)$, for all four quantum systems under consideration. Our results are presented for atoms in Fig. 1, atomic clusters in Fig. 2, nuclei in Fig. 3 and bosons in a trap in Fig. 4. We note that in Fig. 1 we present both the dependence of C on all values of N and separately the dependence of C only on values of N for closed shells atoms. The data of the latter figure is used in the fitting of $\Gamma_{\alpha,\beta}(N)$ to $C(N)$. We also plot linear fitted expressions for S and S_{\max} of the form $S = a + b \ln N$. In order to choose the specific values (α, β) , we employ a new prescription, described below.

We begin with the requirement that the curves $\Gamma_{\alpha,\beta}(N)$ and $C(N)$ should coincide (approximately) or show the same pattern, for a proper pair of values (α, β) . We quantify the

similarity between the two curves with a norm $\sum_{i=1} (C_i - (\Gamma_{\alpha,\beta})_i)^2$, which should be zero in the ideal case, where the two curves are exactly the same. A comparison of $\Gamma_{\alpha,\beta}(N)$ with $C(N)$ has been suggested for the first time in [14], where we obtained by inspection of the figures that roughly $\alpha \simeq 0$ and $\beta \simeq 4$ ($\alpha < \beta$) and has been observed that complexity increases with $N = Z$, based on the trend of closed shells.

We limit our search for the optimal pair (α, β) only for the regions of the $\alpha - \beta$ plane, where the behavior of the complexity measure $\Gamma_{\alpha,\beta}$ is the same with the corresponding behavior of $C(N)$. Thus, if for example, $C(N)$ is a decreasing function of N , then we search for the proper pair (α, β) only in the region where $\Gamma_{\alpha,\beta}$ is decreasing with N . The trend of complexity is dictated by $C(N)$.

The resulting pair of values (α, β) is named *Pair of Order-Disorder Indices (PODI)*. We hope that it will serve as a useful tool in order to classify quantum (or classical) many body systems, according to the contribution of order versus disorder contributions to complexity.

In Fig. 5, we present our results for all four quantum systems under consideration in the $\alpha - \beta$ plane with $(0 \leq \alpha \leq 20$, and $0 \leq \beta \leq 20)$, with mesh points 10^{-2} for α and β . A decreasing behavior is denoted by white color, a convex behavior by grey and an increasing trend by black.

Qualitatively, the structure of the corresponding plots in $\alpha - \beta$ plane and the shape of the complexity regions are the same for all considered quantum systems. A monotonically decreasing region is followed by a convex one and then we have a monotonically increasing area.

Our results are summarized in Table I, where in addition we present the approximate linear limits of the three respective complexity regions (decreasing, convex, increasing).

Of all four quantum systems, atoms and bosons are exceptional, in the following sense.

- Atoms: In atoms there are very hard oscillations of complexity measures with N ($= Z$, atomic number). This is displayed in the corresponding (fourth) plot of Fig. 1, where we employ, just for the sake of comparison, the values $\alpha = 0$ and $\beta = 0.42$, originating from the fitting $C(N) = \Gamma_{\alpha,\beta}(N)$ for closed shells (fifth plot of Fig. 1). It is seen that the overall behavior can not be characterized with certainty as monotonically increasing, decreasing or convex. It is clear that our proposed two-step procedure to find (α, β) and the complexity character i.e. first to observe the general trend of $C(N)$

Quantum System	Trend of Complexity $C(N)$			PODI pair α	Complexity Character
	Decreasing	Convex	Increasing		
Atoms (norm=4.095)	$\alpha \geq 9.09\beta$ & $\alpha \geq 3.33\beta$	$\alpha < 9.09\beta$	$\alpha < 3.33\beta$	0	$\alpha < \beta, \alpha = 0$ Order
Atomic Clusters (norm= 2.88×10^{-5})	$\alpha \geq 2.90\beta$ & $\alpha \geq 1.27\beta$	$\alpha < 2.90\beta$	$\alpha < 1.27\beta$	10.57	$\alpha > \beta$ Disorder
Nuclei (norm= 2.48×10^{-5})	$\alpha \geq 2.47\beta$ & $\alpha \geq 1.25\beta$	$\alpha < 2.47\beta$	$\alpha < 1.25\beta$	10.98	$\alpha > \beta, \beta = 0$ Disorder
Bosons (norm= 5.6×10^{-3})	$\alpha \geq 250\beta$ & $\alpha \geq 15.39\beta$	$\alpha < 250\beta$	$\alpha < 15.39\beta$	120	$\alpha \gg \beta$ Disorder

TABLE I: Analysis of results for the PODI pair (α, β) and complexity character of various quantum systems.

and then fit $\Gamma_{\alpha,\beta}(N)$ to $C(N)$ is not straightforward for atoms, if we insist to employ the dependence of C on all the values of the atomic number $N = Z$. It seems that details of our method should be adjusted, to some extent, to available data and/or the specific system under consideration.

On the other hand, one can find the complexity character based on specific characteristics of the system that may simplify the whole picture (something that can be useful for further studies of complexity especially in classical chaotic systems). In the case of the atoms, such a simplification should be the choice of the closed shells atoms. Thus, we organize our results to define decreasing, convex and increasing behavior using the trend of the closed shells atoms ($Z = 2, 10, 18, 36, 54$). This choice for atoms implies an increasing trend for $C(N)$ and a optimal pair $(\alpha, \beta) = (0, 0.42)$. The same trend has been observed in a series of papers employing various models and methods [13, 14, 16, 30, 31].

- Bosons: The increasing area dominates, while a very narrow decreasing area exists for very small values of β . The optimal pair is $(20, 0.08)$. Extending our calculations in the $\alpha - \beta$ plane for $0 \leq \alpha \leq 120$, and $0 \leq \beta \leq 120$ (we do not display this extension in Fig. 4), we see that the best SDL-LMC fit is attained for $\alpha = 120$ and $\beta = 0.44$.

Regardless of the exact proper value of α , which is certainly larger than 120, we are sure that $\alpha \gg \beta$. This is a clear indication that a bosonic system is to a high degree disordered, as expected from the absence of the Pauli principle.

We observe in Fig. 1-4 and the norms of Table I, that the curves $C(N)$ and $\Gamma_{\alpha,\beta}(N)$ almost coincide for the optimal (α, β) in clusters and nuclei, while for atoms and bosons, they show a similar pattern. This is the best result we can obtain.

It is concluded that three out of the four considered quantum systems (atomic clusters, nuclei and bosons) cannot grow in complexity or organize themselves as the number of particles increases, because $C(N) = \Gamma_{\alpha,\beta}(N)$ is a decreasing function of N . The case of atoms is different, as discussed above. The question is open whether those systems can show self-organization (organized complexity), when they form more complicated structures. At the moment, our indicative result is that for atoms complexity increases with the atomic number. This is interesting and promising for future research, because nature employs atoms as the basic building blocks of molecules and macromolecules related to biology. The next step is obviously to change their environment by influencing them by an external factor. In [12] it was found that increasing the magnetic field applied on excited electrons of atoms, the information content of electrons in specific states is exchanged, while there is simultaneously an energy level avoided crossing of the corresponding states. An other idea is to study the effect of confinement [26], simulating, in a way, the effect of the *environment*.

The most obvious objection to SDL and LMC measures of complexity $\Gamma_{\alpha,\beta}$ and C respectively, is that any function of S e.g. $\Gamma_{\alpha,\beta}(\Delta)$, ($\Delta = S/S_{\max}$) does not give anything new, compared with S . It is still a function of S . This might be true for some cases, but in our specific treatment of quantum many-body systems with N particles, the function $S(N)$ is different than $S_{\max}(N)$, providing a non-trivial dependence of $\Gamma_{\alpha,\beta}(\Delta(N))$ on N .

IV. CONCLUSIONS

It is seen from Table I that for bosonic systems α is very large and β very small, $\alpha \gg \beta$, which implies that they are *disordered*, as expected intuitively, because of the absence of the Pauli exclusion principle. Atoms can be considered as *ordered* ($\alpha = 0$). This seems reasonable and might be expected for a system created by filling with electrons, one by one, well defined orbitals in the Coulomb field of the nucleus. Complexity measures are

oscillating hard in atoms, but our experience leads us to the realization that the proper way to assess complexity is to define the overall trend by observing the closed shells atoms. A similar conclusion has been drawn for atoms in [14] and recently in [16], where $\alpha = 0.085$ and $\beta = 1.015$, obtained with discrete (fractional) occupation probabilities of atomic orbitals, indicating an ordered system. In the same spirit, nuclei and atomic clusters are (less) *disordered* systems ($\alpha > \beta$) lying between atoms and bosons. Thus, a character of order or disorder can be assigned to each system.

In other words, a well specified PODI categorization emerges naturally. Specifically, the inequality $\alpha > \beta$ implies a character of *disorder*, while if $\alpha < \beta$ the system has the character of *order* and there is the possibility of gradual change of their character from order to disorder realized in the chain Atoms-Atomic Clusters-Nuclei-Bosons (from left to right), or equivalently from top to bottom in Table I. It is seen in Fig.6 that nuclei and bosons lie very close to the α -axis (disorder), atoms on the β -axis (order), while for atomic clusters the disorder character is much stronger than that of order.

The proper values of (α, β) are obtained by an overall fitting of $\Gamma_{\alpha,\beta}(N)$ with two free parameters to a single curve $C(N)$ (no free parameters). Hence our statement that a quantum system is ordered or disordered comes from an overall comparison of the corresponding values of α and β , i.e. for a range of values of N . This enables us to find the optimal pair (α, β) . If one wishes to obtain values (α, β) imposing the condition $\Gamma_{\alpha,\beta}(N) = C(N)$, for a fixed value of N , then one has to fit a two parameter formula $\Gamma_{\alpha,\beta}(N)$ to just a single value $C(N)$, obtaining a dependence $\beta(\alpha)$. We also observe that for all systems studied in this work the functional of disequilibrium $D(N)$ is very similar to $\Gamma_{\alpha,\beta}(N)$ and $C(N)$. The same observation was made in [16]. Thus a new measure of complexity emerges.

The usefulness of our approach may be validated pragmatically, by future applications of PODI to several other quantum and classical systems. First, it should give reasonable results appealing to intuition. Second and more important, it is interesting to investigate cases where, instead of varying just the number of particles N , one might study the effect on complexity of other relevant quantities of the systems as well. For example, the next step may be to study explicitly the effect of correlations between particles and/or the influence of various theoretical models of the systems.

It is noted that so far there is no single or perfect quantitative definition of complexity. It is quite natural that any effort to provide a specific definition will be met with criticism

[27, 28, 29]. Comments on the validity of SDL and LMC measures can be found in Sec. 4 of [14], where welcome properties of any definition of complexity are described in detail.

An answer to the question whether complexity shows an increasing or decreasing trend with N , is dictated by the behavior of $C(N)$. It is seen, that, at least for the models employed in the present paper, the trend of complexity is clearly decreasing for atomic clusters, nuclei and bosons, while it is increasing for atoms. An additional merit of the present work is the proper choice of the strength of disorder parameter α and the strength of order β , by imposing the approximate condition $\Gamma_{\alpha,\beta}(N) = C(N)$ for all values of N . This is in accordance with a relativistic treatment of atoms, where $C(N)$ shows an increasing behavior as well [30, 31]. However, in our opinion, the latter result is not conclusive, because the corresponding studies are taking into account only the density distribution in position-space $\rho(\mathbf{r})$. A proper, complete treatment of information entropy S and disequilibrium D should take into account both position- and momentum-spaces, indicated by relations $S = S_r + S_k$ and $D = D_r \cdot D_k$, employed in our present calculations.

So far, (α, β) indices have been found for quantum systems. The same PODI procedure can be applied to quantify order versus disorder in classical systems, provided that they can be treated probabilistically. Such an application might be the use of probability densities in chaotic mappings, enabling researchers to quantify and observe the change of strengths of disorder α and order β , while non-linear systems evolve through various phases of chaotic behavior and routes to chaos. Finally, one may, in principle associate to any probability distribution depending at least on one or more parameters describing a system, a PODI pair and a corresponding complexity character, under certain conditions. This case can be examined in a future work.

The present study focuses on the dependence of complexity on just one parameter, i.e. the number of particles. The same method might be extended to assess the effect of other relevant quantities, which are of interest in the context of a specific level of description of a system. Another point, related to the latter one, is that in order to answer a specific question about organized complexity and the interplay of order versus disorder, one might need a sophisticated model of the dynamics of the system, or, in some cases, a simplified one, describing only salient but necessary features. This might be sufficient for a special aim. From this point of view, the merit of a model describing a system will be assessed, not only by its ability to reproduce exactly experimental results or simulate its behavior accurately,

but by being robust enough to uncertainties or errors in description. This is facilitated by the fact that an inequality between α, β can be satisfied for a wider range of values, instead of an approximate equality $\alpha \simeq \beta$.

In brief, we propose a recipe to find a Pair of Order-Disorder Indices (α, β) (PODI) in order to classify the order versus disorder contribution to complexity of any system (in our case quantum) described probabilistically. This enables us to assign to systems the category of order ($\alpha < \beta$) or disorder ($\alpha > \beta$) and classify them accordingly.

We conclude that the atom is the only system out of four, studied here, which is *ordered and growing in complexity with the number of particles (electrons)*. We conjecture that this is an information-theoretic explanation, supporting the fact that atoms are used by nature, as suitable components to construct larger structures i.e. molecules and macromolecules. It is also a first step, with encouraging results, to fulfill quantitatively Wheeler's plan as the fundamental theory of the Cosmos.

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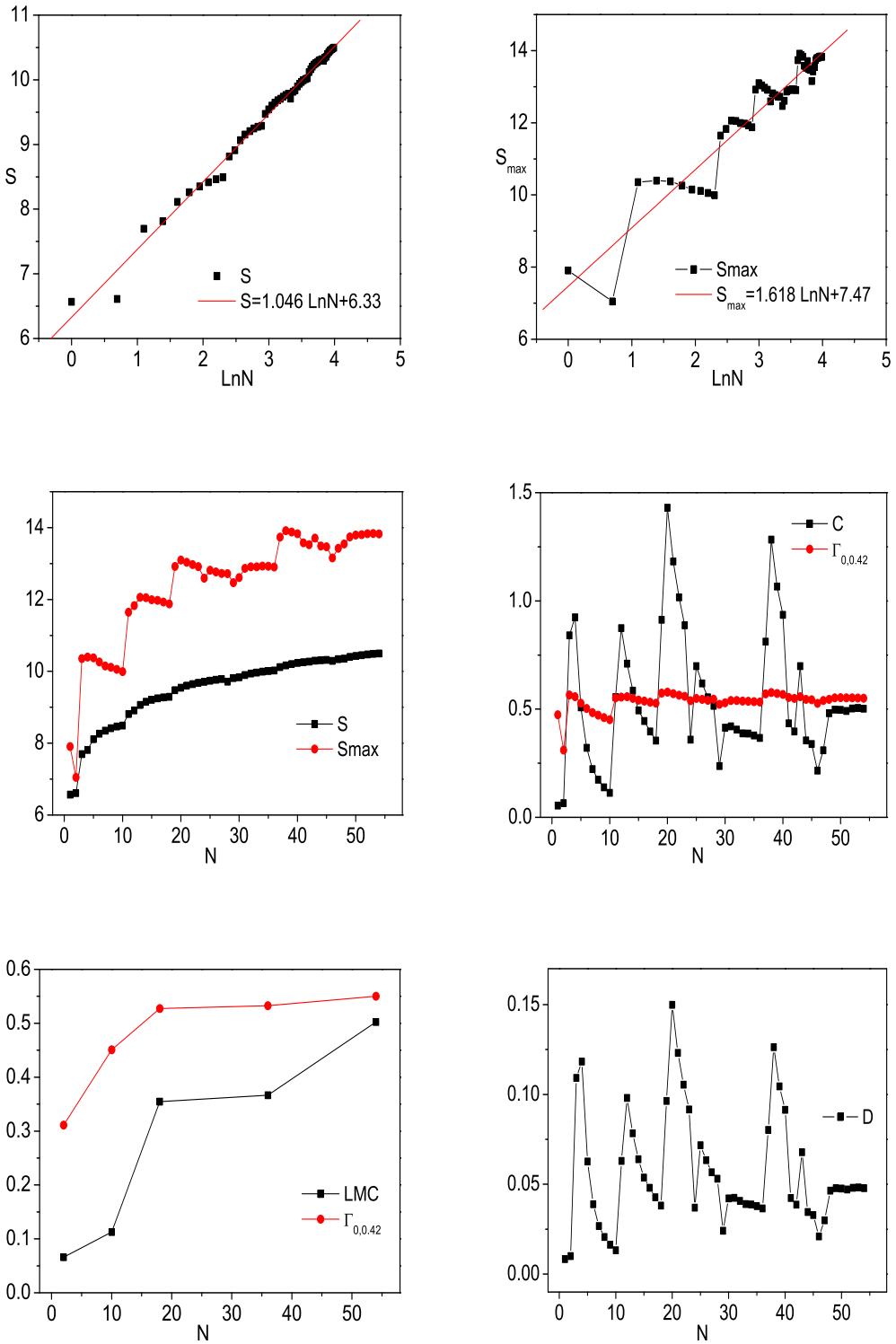


FIG. 1: Atoms

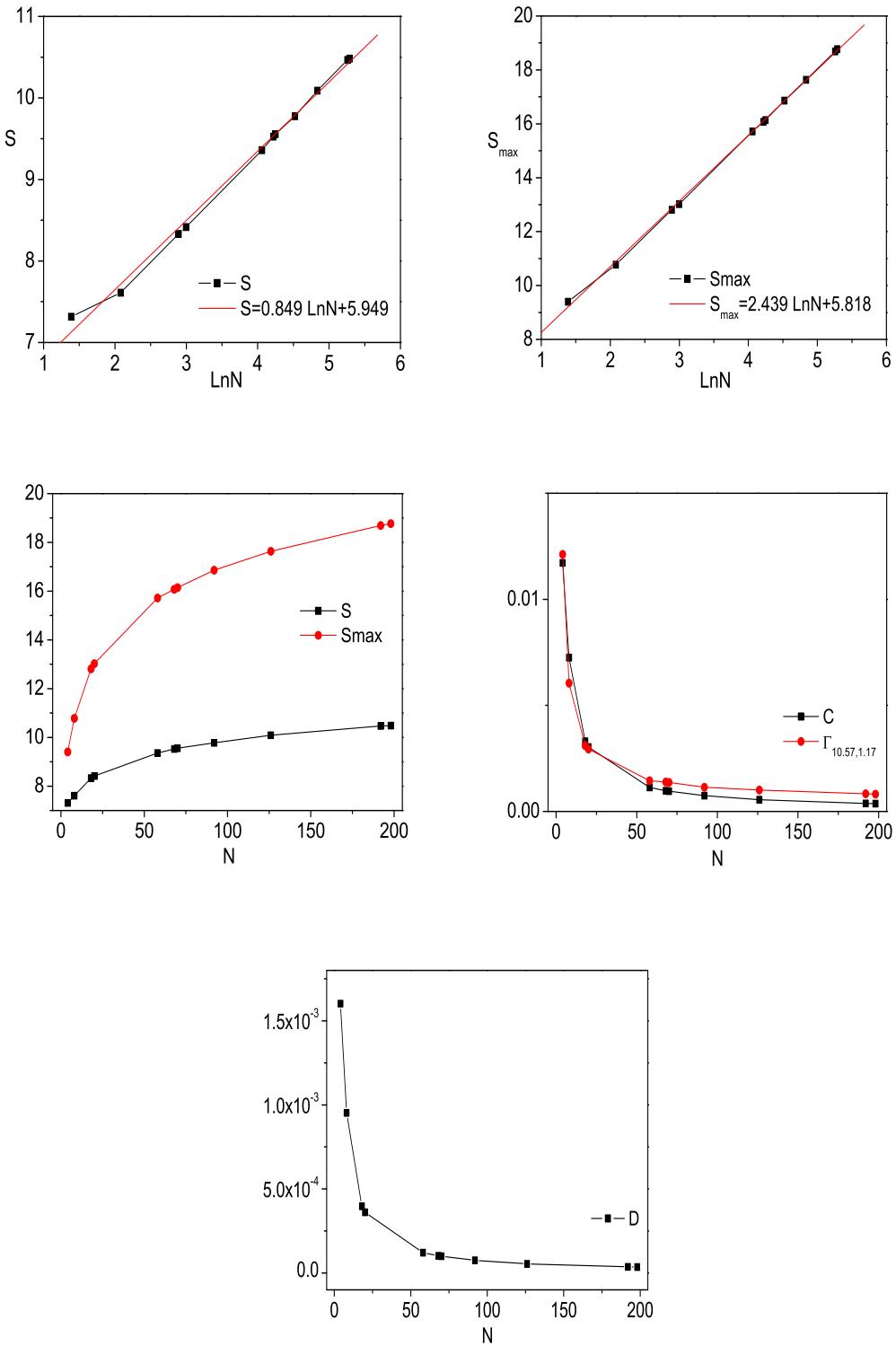


FIG. 2: Atomic Clusters

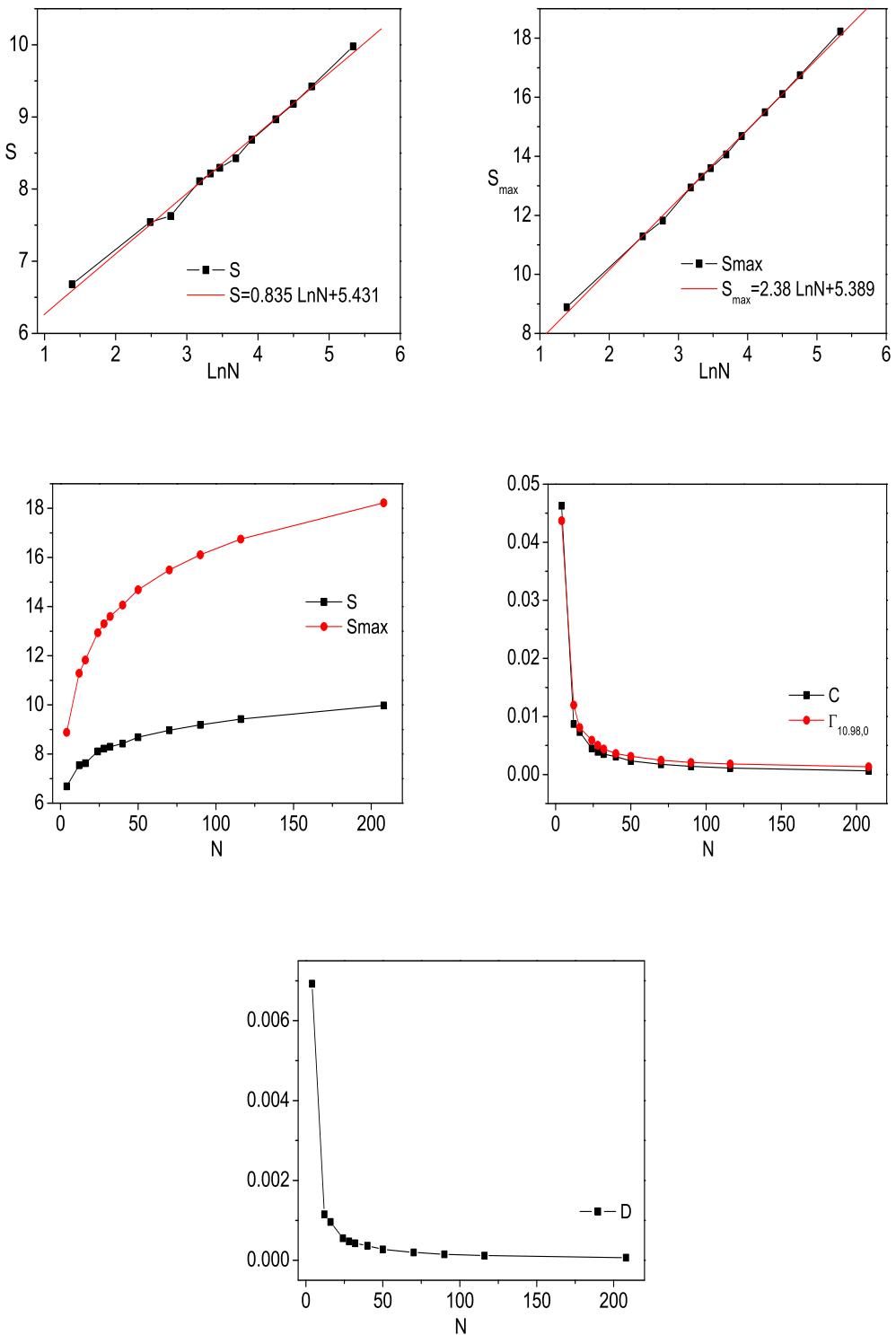


FIG. 3: Nuclei

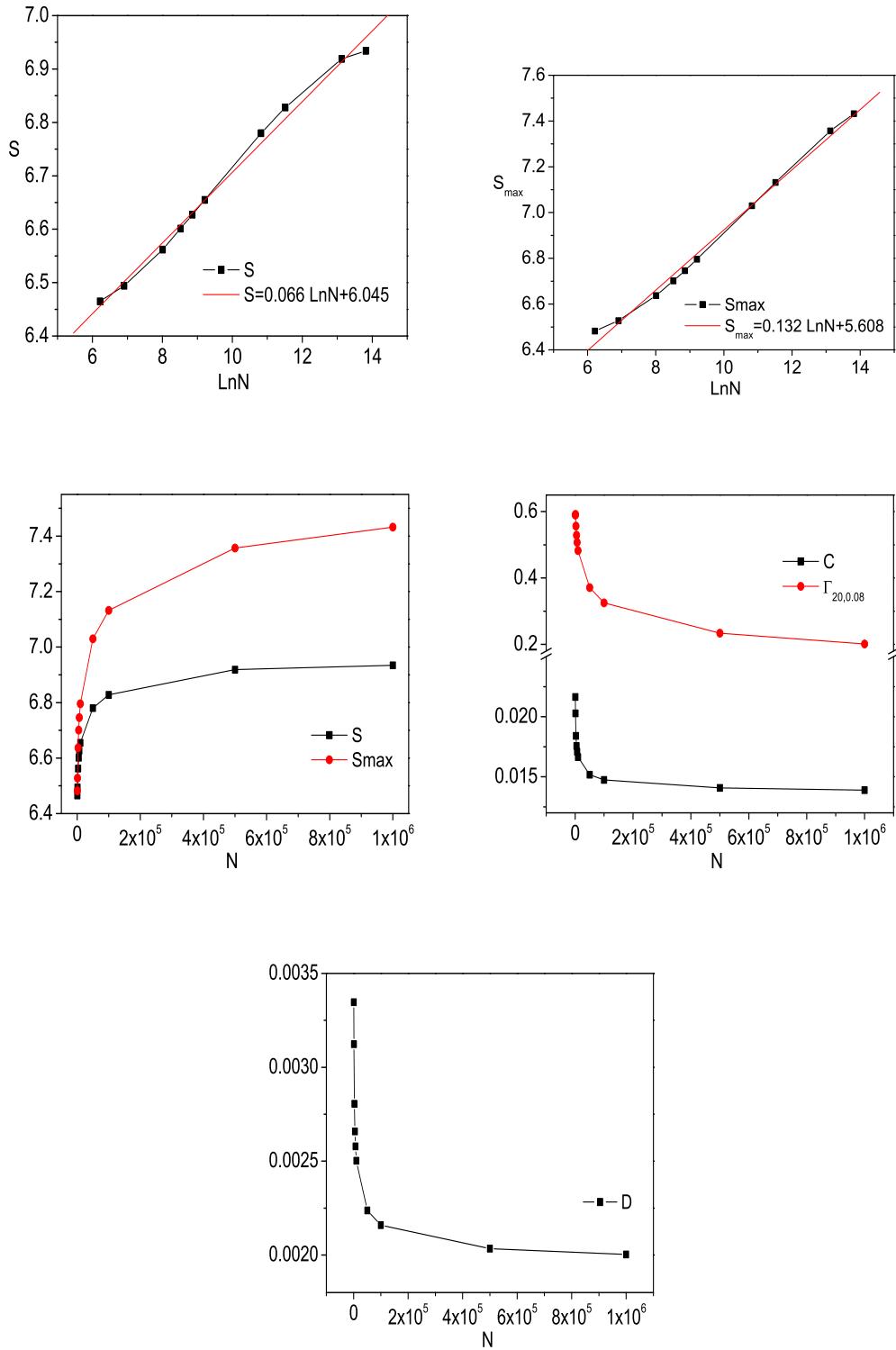


FIG. 4: Bosons

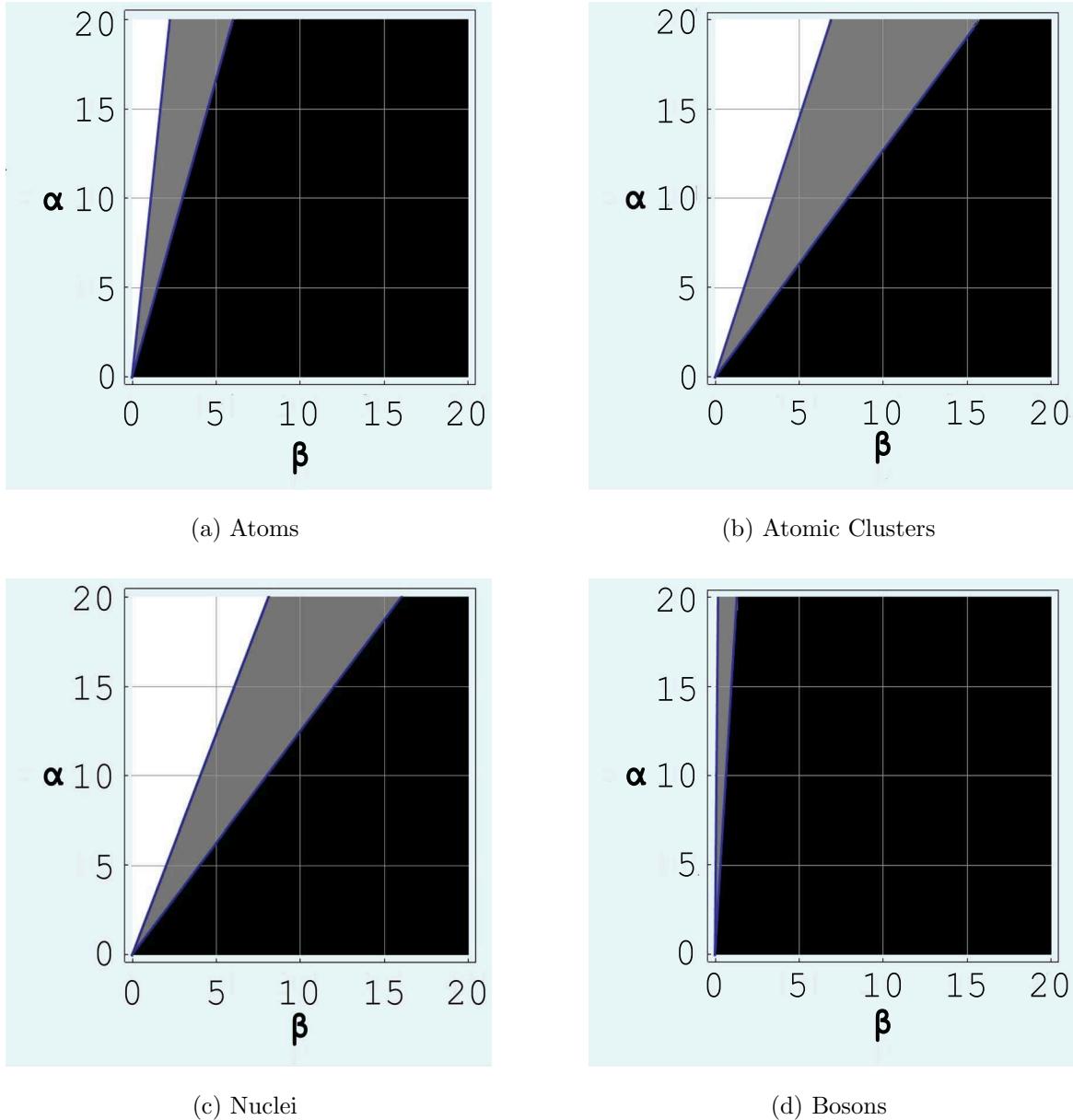


FIG. 5: Complexity regions in the $\alpha - \beta$ plane, showing a decreasing trend of $\Gamma_{\alpha,\beta}(N)$ (white), a convex (grey) and an increasing one (black), for (a) atoms, (b) atomic clusters, (c) atomic nuclei, and (d) bosons. The approximate linear boundaries of the three regions are given in Table I.

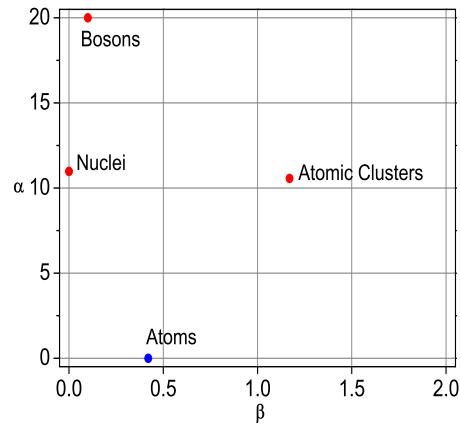


FIG. 6: PODI values (α, β) for atoms, atomic clusters, nuclei and bosons. The α -axis corresponds to "disorder" while the β -axis to "order".

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